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Pigs aplenty

GambleAware

NSW GOVERNMENT

December 2022

Activity introduction

Quick summary

‘Jeopardy’-style games are games where you must decide whether or not to risk jeopardising your position in return for greater gains. Pig is one example, a very simple dice game dating back to the 1940s. There are many different versions of it. We will use the single-die version, but it can also be played with two dice, cards, or even plastic molded pigs!

This lesson will have students exploring the concept of pushing their luck, and developing a strategy to help them decide when to quit. They will then reflect on how these strategies help people to partake in gambling in a way which minimises their risk of experiencing harms.

Learning intentions

Students will:

- understand that optimal strategies often involve quitting while you're ahead
- understand the concept of risk vs. reward.

21st-century skills

Communicating

Creative thinking

Critical thinking

Digital literacy

Entrepreneurship

Ethical behaviour

Personal and social skills

Problem solving

Teamwork

Syllabus outcomes

Probability 1 - Statistics and Probability

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-21SP** represents probabilities of simple and compound events.

Number and Algebra - Fractions, Decimals and Percentages

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-5NA** operates with fractions, decimals and percentages.

Life Skills (Mathematics)

- **MALS-38SP** recognises and uses the language of change in a range of contexts
- **MALS-39SP** recognises the elements of chance and probability in everyday events
- **MALS-8NA** recognises and compares fractions in everyday contexts
- **MALS-9NA** represents and operates with fractions, decimals or percentages in everyday contexts.

General capabilities

Numeracy

Critical and creative thinking

Ethical understanding

Relevant parts of Year 7 and 8 achievement standards

Students solve problems involving percentages and all four operations with fractions and decimals. They compare the cost of items to make financial decisions. Students represent numbers using variables. Students use fractions, decimals and percentages, and their equivalences. They express one quantity as a fraction or percentage of another. Students determine the sample space for simple experiments with equally likely outcomes and assign probabilities to those outcomes.

Students solve everyday problems involving rates, ratios and percentages. Students solve problems involving profit and loss. They choose appropriate language to describe events and experiments. Students determine the probabilities of complementary events and calculate the sum of probabilities.

Stage 4 Mathematics Syllabus Statement

Students use mathematical terminology, algebraic notation, diagrams, text and tables to communicate mathematical ideas, and link concepts and processes within and between mathematical contexts. They apply their mathematical knowledge, skills and understanding in analysing real-life situations and in systematically exploring and solving problems using technology where appropriate. Students operate competently with integers, fractions, decimals and percentages, and apply these in a range of practical contexts. Students calculate the probability of simple and complementary events in single-step chance experiments.

Topic
Gambling probability

Unit of work
Mathematics Stage 4

Time required
60 minutes

Level of teacher scaffolding
High - students will require strong scaffolding through the explicit instruction on calculating probabilities, but will be able to perform the tasks independently.

Resources required

- Calculators – one per student
- Six-sided die – one per pair of students
- Student workbooks

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming.

Teacher worksheet

Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers read the Facilitator Pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Learning intentions

Students will:

- understand that optimal strategies often involve quitting while you're ahead
- understand the concept of risk vs. reward.

Success criteria

Students can:

- convert between probabilities and odds
- critically evaluate the concept of risk vs. reward in a given situation
- develop strategies to improve the likelihood of winning.

Teaching sequence

15 minutes - Part A: Greedy pigs

20 minutes - Part B: Smart pigs

20 minutes - Part C: Brilliant pigs

5 minutes - Reflection

Part A: Greedy pigs

Work through this resource material in the following sequence:

Step 1

Ask your students if they have ever been in a situation where they should have quit when they were ahead. Something where afterwards they thought, 'if only I'd stopped earlier.'

Some students might volunteer to share their experiences.

Step 2

Explain that a lot of people experience gambling harms when they continue betting long after they should have stopped because they were risking more than they could afford to lose. This often comes from 'chasing losses'. That's where you keep gambling in order to get back the money you have lost. But since every bet is a losing proposition on the basis of probability, chasing losses just winds up with more money being lost.

Step 3

Explain to students how to play the classic push-your-luck game, Pig:

1. On a player's turn they roll the die and add the value to their 'turn total'. Their turn ends when:
    a. They roll a 1, and their turn total for that turn is wiped out to zero. However, their overall score remains the same.
    b. They choose to 'hold' or 'bank', and add their turn total to their overall score.
2. Once their turn is over, play passes to the next player.
3. The first player to reach 100 points is the winner.

For example, say the first player rolls a 4 and chooses to continue. They roll a 3. Their turn total is now 7. They again choose to continue. They next roll a 1. Their turn ends, and they add nothing to their total overall score.

The second player rolls 2, 3, 6, 2, and chooses to hold. They add 13 to their total overall score.

Step 4

Divide your class into pairs, or allow students to choose a partner. Give each pair a six-sided die. They will need paper and pens in order to keep score. Have your class play a few games of Pig. Ask students to think closely about when (after how many rolls) they are deciding to stop rolling and bank their score, or what turn score they feel like they have to achieve before they're comfortable with ending their turn, or any other factors that seem to be influencing their play.

Step 5

After students have finished their games, ask whether any students developed a strategy. Did this seem to improve their chances of winning?

Have some students volunteer their strategies and thought processes that developed during the games, focusing on drawing out the winning strategies.

Part B: Clever pigs

Step 1

Explain to your class that the game of Pig has been mathematically 'solved'. Computer scientists have discovered that there is a mathematical way to decide the 'best play' on any given roll of the dice, whether to keep rolling, or hold. Ask your class what factors they think would influence that decision. There are three major ones:

- your own overall score
- your opponent's overall score
- your current turn total.

Step 2

Explain that best play in this case still relies on chance. Some skill-based games, such as Connect Four, have been solved in such a way that a player making the best moves will always win. Other games, like Blackjack, have been solved to give the player the best chance, but not a guaranteed win.

Note: Playing Blackjack perfectly does not give you an advantage over the casino. It lowers the casino's advantage from around 2% to 0.5%, which still results in a net loss over time for the gambler.

More complicated games like Chess and Go have not yet been solved, and in fact it is not yet certain if they are even solvable.

Pig is more like Blackjack in this case: the best move can be suggested, but the luck of the dice could still undo your great strategy.

Step 3

Independently, in their pairs, students work to come up with a possible strategy.

Let them know that the actual mathematical solution is very complicated, and would require a computer program to tell you what to do. Right now we just want a basic strategy to give you a slight advantage.

Prompt student thinking by asking:

- When does it feel right to stop rolling and bank your turn score?
- Do you think there is a certain number of dice rolls after which you should stop rolling and bank your turn score?
- Do you think there is a certain turn score you should reach, after which you should stop rolling and bank your turn score?

Step 4

Have a class discussion, allowing each pair to share their ideas on strategy with the class.

Step 5

Have your class play another round of Pig, this time using their own strategy. Ask them how confident they feel before playing. How many games do they think would be needed to see if their strategy is any good? That is, how much data would they need to collect to 'prove' their strategy?.

Part C: Brilliant pigs

Step 1

Explain to your class that one way of thinking of the game is that every time you roll, you are making a bet that you will not get a 1. The probability of this is:

$$P\,(\textit{Not rolling a } 1) = \frac{5}{6}$$

In other words, the probability of increasing your turn total is 5/6.

Step 2

Ask your class if you should roll when your turn total is zero. You should, because at this stage you have nothing to lose. Now ask them if you should roll if your turn total was 40. That's a pretty high score, so most people would probably advise banking your score.

Step 3

Let's now calculate the expected value of a roll. Say your current turn total is 11 and you have decided to keep rolling. There are six possible outcomes:

1. You **lose** 11 points
2. You **gain** 2 points
3. You **gain** 3 points
4. You **gain** 4 points
5. You **gain** 5 points
6. You **gain** 6 points.

Each of these has a probability of 1/6.

We can use this information to calculate the expected value of this roll:

Outcome	Probability of outcome	Product
-11	$\frac{1}{6}$	$-11 \times \frac{1}{6} = -\frac{11}{6}$
2	$\frac{1}{6}$	$2 \times \frac{1}{6} = \frac{1}{3}$
3	$\frac{1}{6}$	$3 \times \frac{1}{6} = \frac{1}{2}$
4	$\frac{1}{6}$	$4 \times \frac{1}{6} = \frac{2}{3}$
5	$\frac{1}{6}$	$5 \times \frac{1}{6} = \frac{5}{6}$
6	$\frac{1}{6}$	$6 \times \frac{1}{6} = 1$
Expected value (sum of products):		$\frac{3}{2} = 1.5$

We can see here that the expected value of rolling now is 1.5 points, meaning you should take the risk.

Step 4

Have your students calculate the expected value if the turn total was 24.

Outcome	Probability of outcome	Product
-24	$\frac{1}{6}$	$-24 \times \frac{1}{6} = -4$
2	$\frac{1}{6}$	$2 \times \frac{1}{6} = \frac{1}{3}$
3	$\frac{1}{6}$	$3 \times \frac{1}{6} = \frac{1}{2}$
4	$\frac{1}{6}$	$4 \times \frac{1}{6} = \frac{2}{3}$
5	$\frac{1}{6}$	$5 \times \frac{1}{6} = \frac{5}{6}$
6	$\frac{1}{6}$	$6 \times \frac{1}{6} = 1$
Expected value (sum of products):		$-\frac{2}{3} = -0.67$

The expected value is a loss, so this suggests that rolling with a turn total of 24 is not a good idea.

Step 5

Can we use this to determine the point at which the expected value changes from negative to positive? Have your class calculate the expected value if your turn total is *t*.

Outcome	Probability of outcome	Product
t	1/6	$-t \times 1/6 = -t/6$
2	1/6	$2 \times 1/6 = 1/3$
3	1/6	$3 \times 1/6 = 1/2$
4	1/6	$4 \times 1/6 = 2/3$
5	1/6	$5 \times 1/6 = 5/6$
6	1/6	$6 \times 1/6 = 1$
Expected value (sum of products):		$(20-t)/6$

The expected value will be positive whenever $(20-t)/6 > 0$, or when $t < 20$.

Step 6

All of this means that a simple strategy for Pig is to always keep rolling as long as your turn total is less than 20.

Step 7

Ask your class if they can think of any exceptions to this strategy.

The two biggest ones are:

- Your opponent is way ahead of you
- You are way ahead of your opponent.

In these cases you should either tighten or loosen up. Imagine your opponent has an overall score of 99, and it's your turn. Your overall score is 78, and your turn total is 20. Your opponent will probably win next turn, so you should push your luck and go for victory.

If you are significantly ahead of your opponent, you should tighten your play. Go for smaller gains and hold at turn totals less than 20.

Step 8

Have your class play a final round of Pig, one person using this strategy, and the other doing what feels right. Afterwards, tally up the wins for those who used the strategy.

Did the strategists win over half the games?

Reflection

As a class, hold a discussion:

- How difficult was it for students to avoid having 'just one more turn', even when they knew it was a bad idea?
- When students were further behind their opponent, were they more willing to risk their luck, even when their turn score crept above 20?
- When students did risk another roll even when their turn score was above 20, how did they feel when this risk paid off and they increased their score?
- Conversely, how did they feel when they took a risk and saw all their hard won points reduced to 0?

Explain to students that there are ways to gamble with reduced risk, and sometimes making bets on less than ideal odds can pay off. However, on average, gambling results in losses over time. Gamblers can feel compelled to 'chase their losses', even when the odds are against them.

Ask students to write a short paragraph reflecting on their approach to pushing their luck and games of chance, and knowing the likelihood of winning and losing.

What impact might knowing the odds have on their behaviour? Will they continue to gamble on a big win, or will they realise this might not be a rewarding course of action?

Teacher reflection

Take this opportunity to reflect on your own teaching:

What did you learn about your teaching today?

What worked well?

What didn't work so well?

What would you share?

Where to next?

How are you going to get there?