Assessment task 2

Non-transitive dice

GambleAware

NSW GOVERNMENT

February 2024

Activity introduction

Quick summary

Non-transitive dice (or intransitive) dice work similarly to the game *Rock, Paper, Scissors*. Three dice, A, B, and C, can be designed such that A will (over half the time) beat B, B will beat C, and C will beat A.

This assessment includes a number of questions on the Appendix A: Student assessment, as well as the answers as part of the teacher instructions, to enable students to demonstrate their understanding and learning across this unit. Students will be asked to explore the outcomes of a set of non-transitive dice using probability tree diagrams, and discover their unique features.

Learning intentions

Students will:

- understand that simple dice can have surprising features
- understand how tree diagrams can be used to explore outcomes.

Success criteria

Students can:

- use tree diagrams to determine the probability of different events
- manipulate the initial conditions of events to adjust their probabilities.

Syllabus outcomes

- **MAO-WM-01** develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly
- **MA5-PRO-C-01** solves problems involving probabilities in multistage chance experiments and simulations
- **MA5-PRO-P-01** solves problems involving Venn diagrams, 2-way tables and conditional probability.

The identified Life Skills outcome that relates to this activity is **MALS-PRO-01** applies chance and probability to everyday events.

Capabilities and priorities

Literacy
Numeracy
Critical and creative thinking

Topic
Gambling probability

Unit of work
Mathematics Stage 5

Time required
60 minutes

Resources required

- Appendix A: Student assessment
- Appendix B: Assessment rubric
- Calculators – one per student
- Dice – 3 per student
- Stickers

Keywords
Gambling, betting, sports, casino, money, wellbeing, gaming, probability tree diagram.

Teacher worksheet

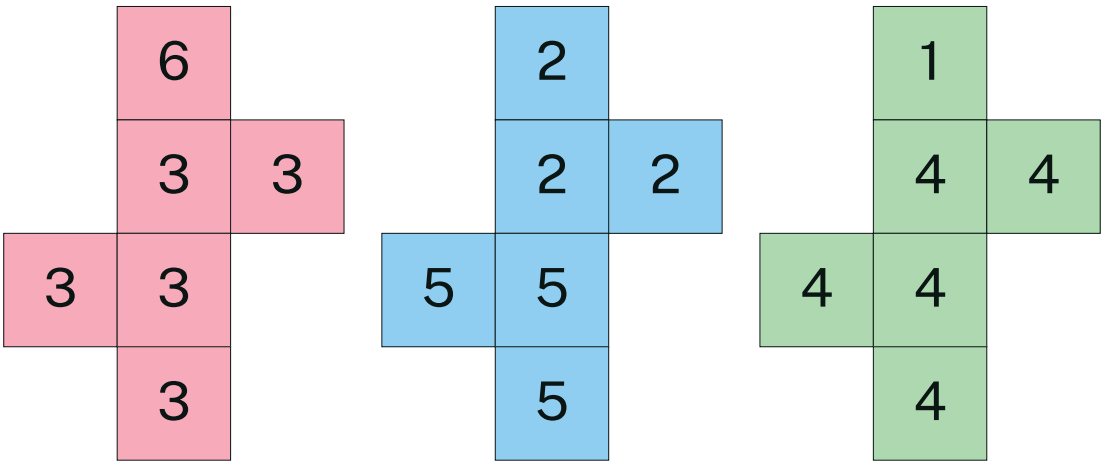
Teacher preparation

Gambling can be a high-risk activity and is a priority concern for young people. Therefore, before conducting the lesson on gambling, it is recommended that teachers read the Facilitator pack. The pack provides teachers and parents with essential information about gambling harm amongst young people and clarifies the nature of gambling-related behaviours and how to approach sensitive topics.

Work through this resource material in the following sequence:

Give each student a copy of Appendix A: Student assessment and ask them to work through the tasks. Support them as necessary. Answers have been included below, *shown in italics*, with specific explanations for each of the questions.

Instructions:



Imagine a simple dice game where two players choose a different coloured dice from the ones shown above. They roll them ten times and score a point each time they get the higher number. No two dice share numbers so it is impossible for a round to result in a draw.

Task 1

Write a prediction for which die you think has the best chance of winning, and explain your thinking.

Task 2

Calculate the average result for each dice.

They are all 3.5

Task 3

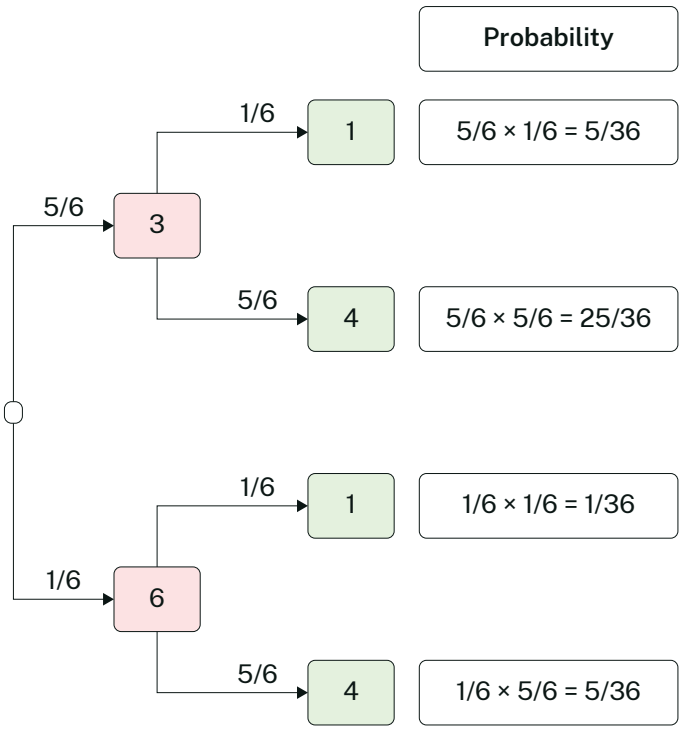
Probability tree diagrams are similar to tree diagrams, but they include the probability of each outcome written on the branch.

What are the probabilities of each possible outcome of rolling two dice?

Create three probability tree diagrams (red blue, red green, blue green) to find the answer.

Show the probabilities on each of the lines where indicated, then record the winner, and the probability of each outcome.

The first one has been done for you.

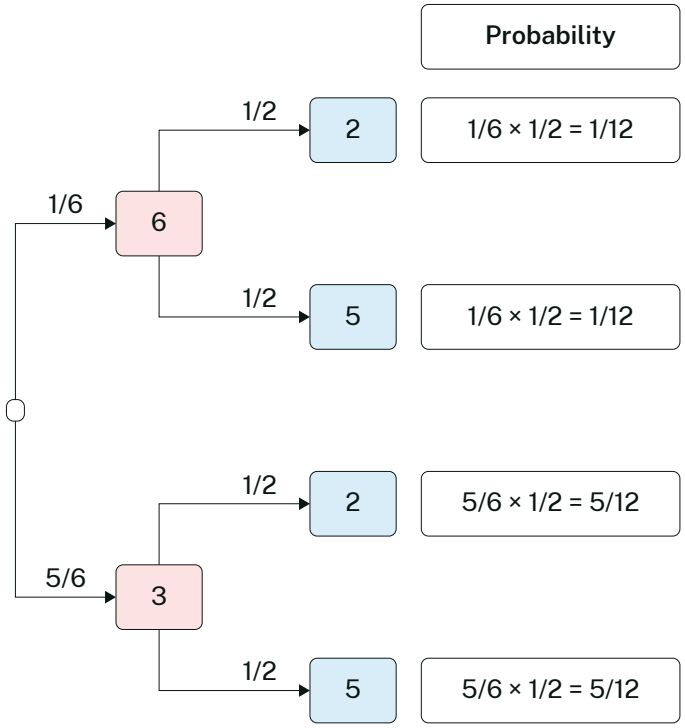


P(green win) = 25/36 ≈ 0.694

P(red win) = 11/36 ≈ 0.306

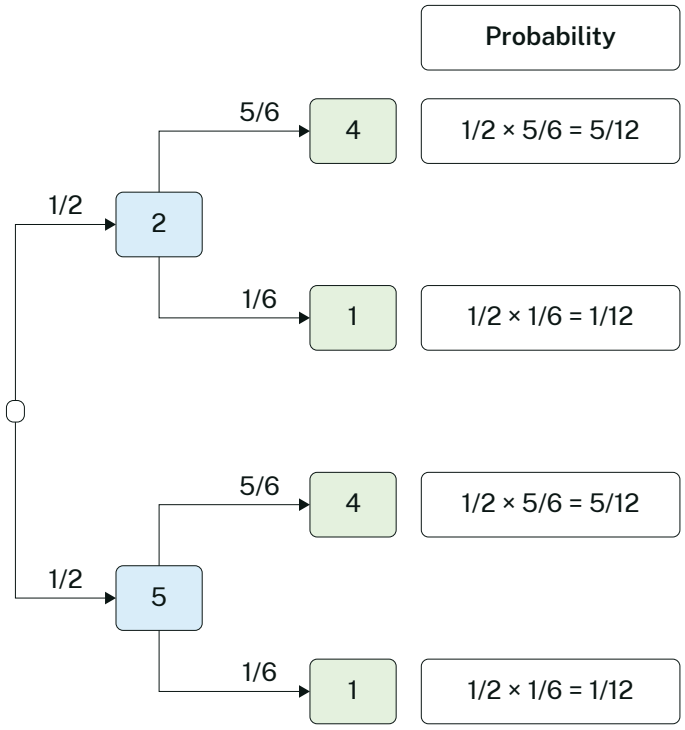
Hence for the red dice there are only 2 branches needed, one for 3 with the probability ⅚ written on it, and one for 6 with the probability ⅙ written on it.

The diagrams should show the win/lose probability of each pair.



P(red win) = 7/12 ≈ 0.583

P(blue win) = 5/12 ≈ 0.417



P(blue win) = 7/12 ≈ 0.583

P(green win) = 5/12 ≈ 0.417

Task 4

Write down your observations of these outcomes.

Red has a higher chance of winning against blue.

Blue has a higher chance of winning against green.

Green has a higher chance of winning against red.

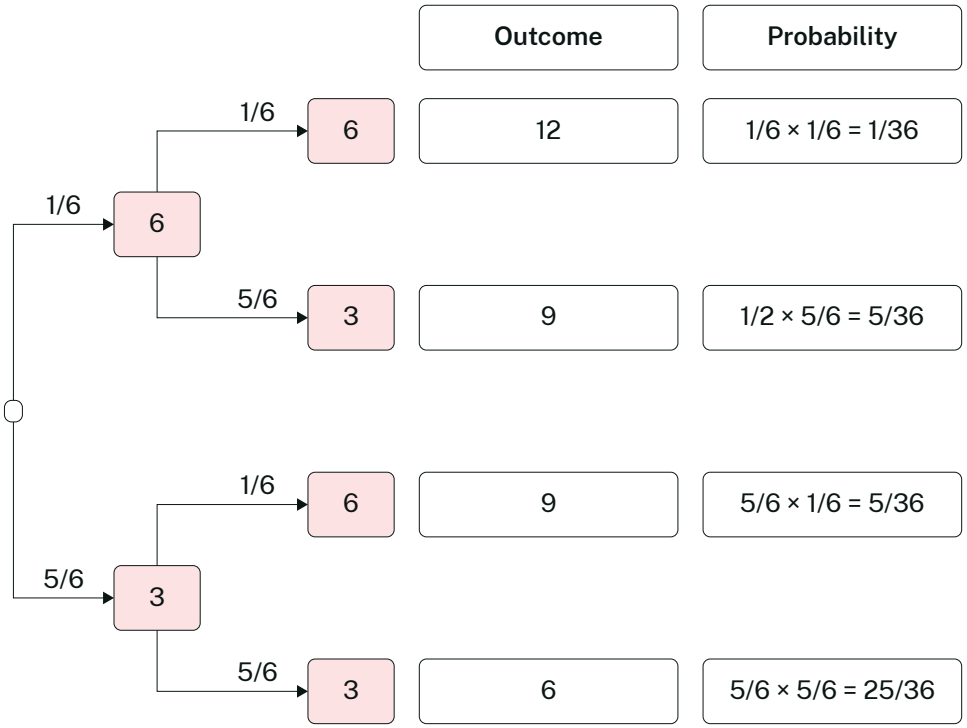
The fact that these dice work in a similar way to 'Rock, Paper, Scissors' is what makes them 'non-transitive.'

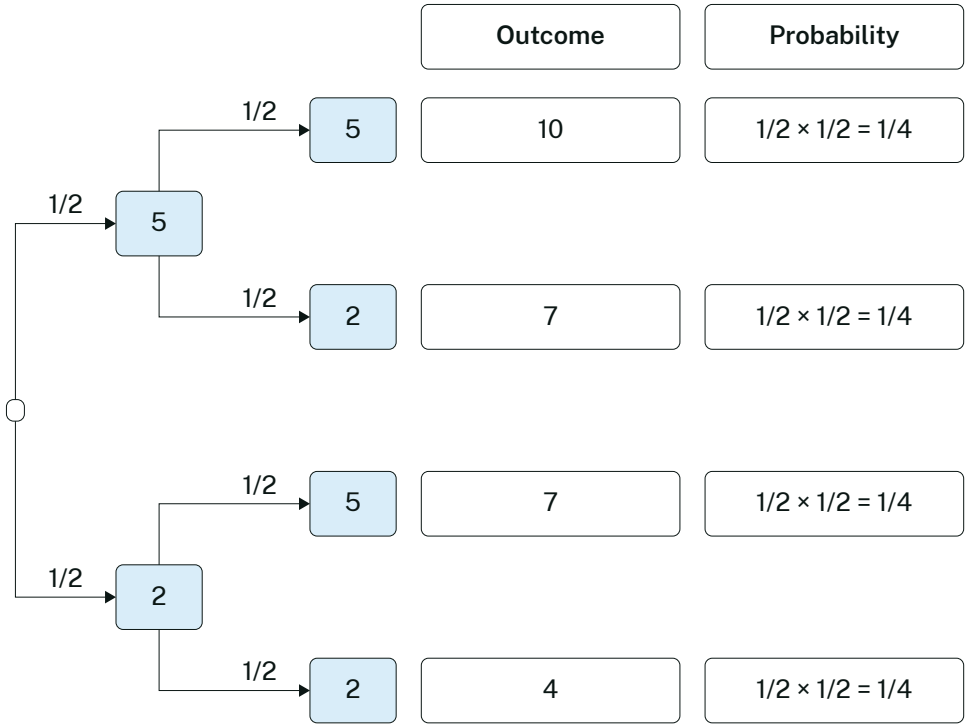
Task 5

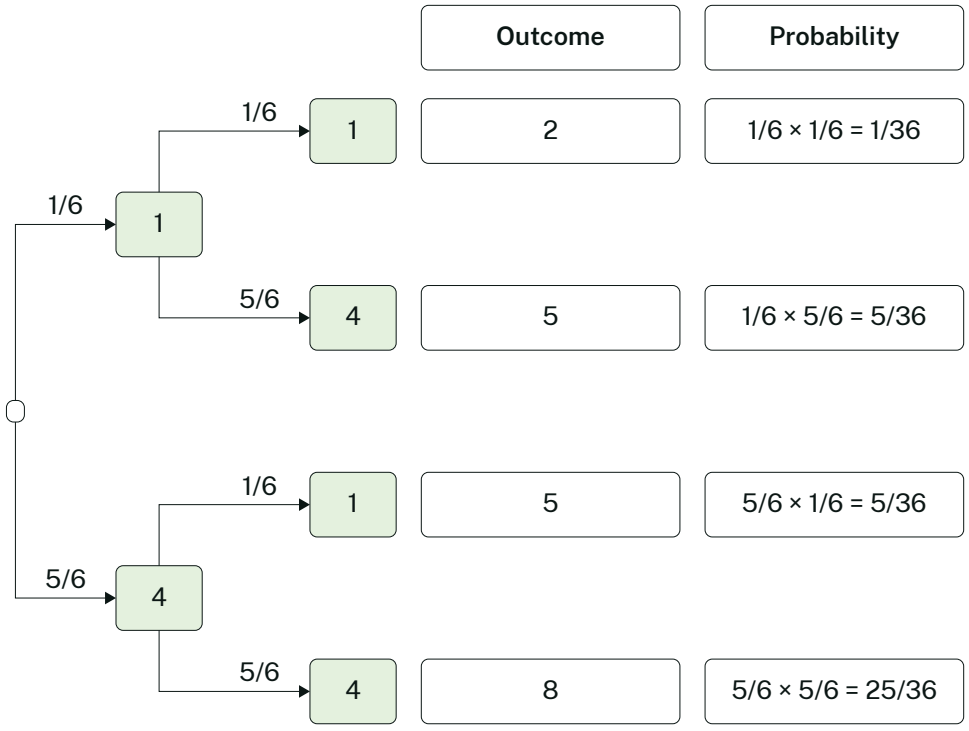
What would happen if the rules of the game changed so that you rolled two dice of each colour?

Create tree diagrams showing the results of rolling two red, two blue, and two green dice.

The first one has been done for you.

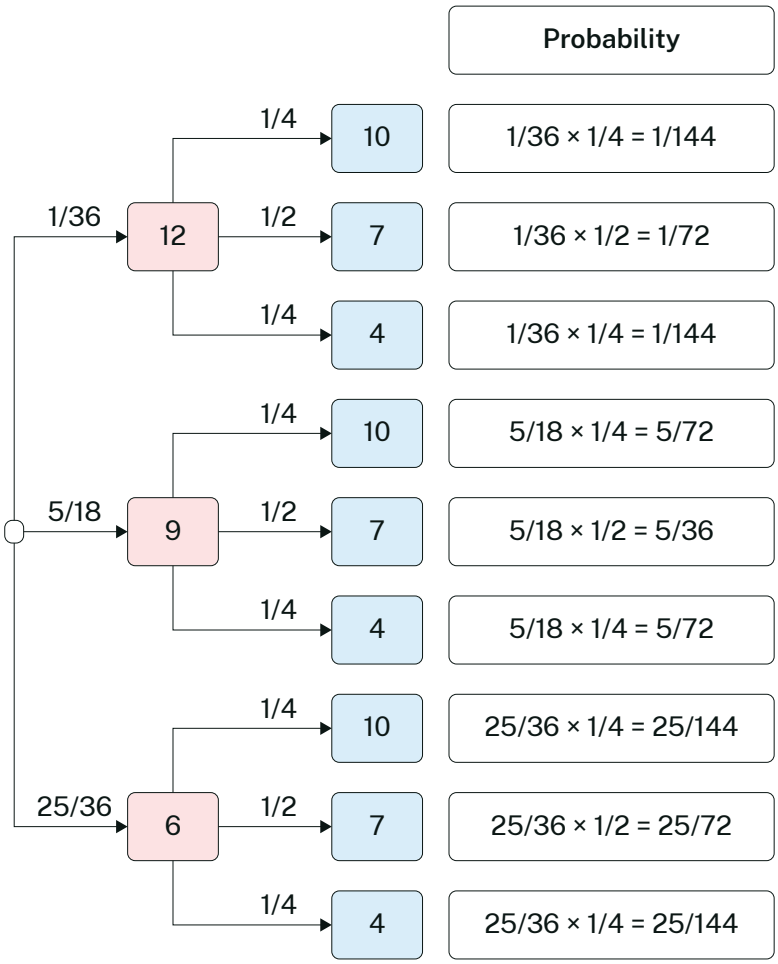



Outcome
Probability
1/2
5
10
1/2 × 1/2 = 1/4
1/2
5
1/2
2
7
1/2 × 1/2 = 1/4
1/2
5
7
1/2 × 1/2 = 1/4
1/2
2
1/2
2
4
1/2 × 1/2 = 1/4


Outcome
Probability
1/6
1
2
1/6 × 1/6 = 1/36
1/6
1
5/6
4
5
1/6 × 5/6 = 5/36
1/6
1
5
5/6 × 1/6 = 5/36
5/6
4
5/6
4
8
5/6 × 5/6 = 25/36

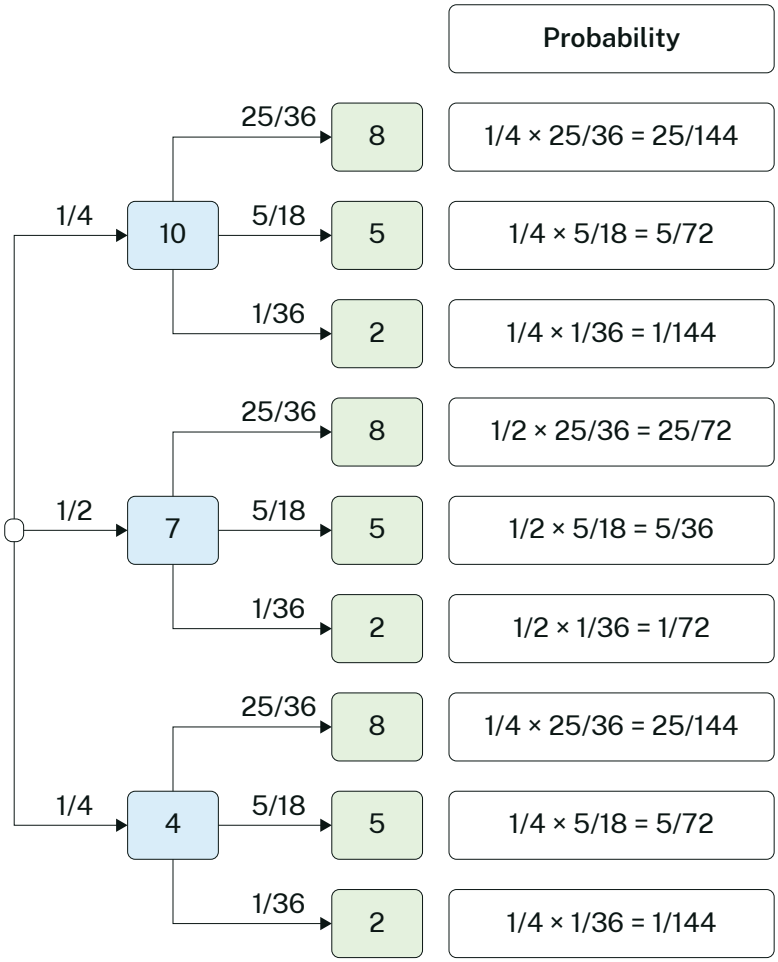
Task 6

Now that we know the probable outcomes of each roll, create a final set of three tree diagrams showing the outcomes of a match between each pair of two dice.



P(red win) = 59/144 = 0.410

P(blue win) = 85/144 = 0.590



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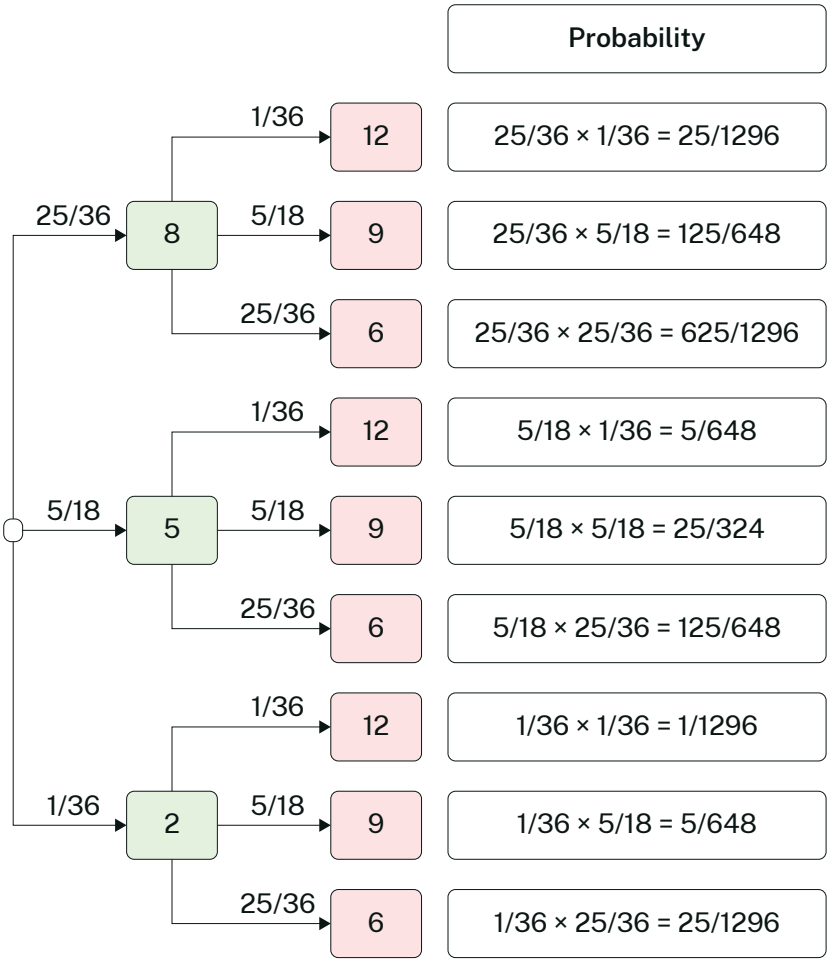


P(green win) = 625/1296 = 0.482

P(red win) = 671/1296 = 0.518

Task 7

Write down your observations of these outcomes.

Blue has a higher chance of winning against red.

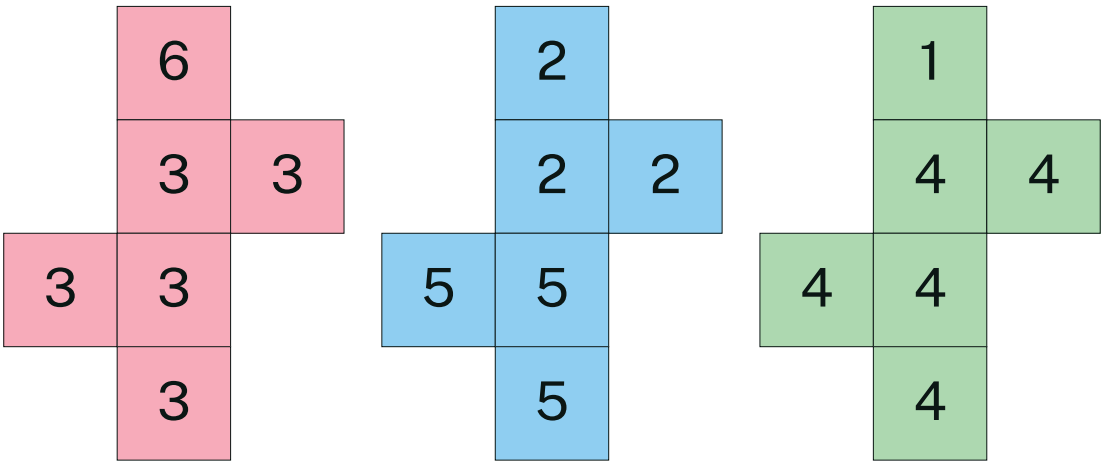
Green has a higher chance of winning against blue.

Red has a higher chance of winning against green.

The likelihood of each coloured dice winning has reversed.

Appendix A: Student assessment

Instructions:



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Calculate the average result for each dice.

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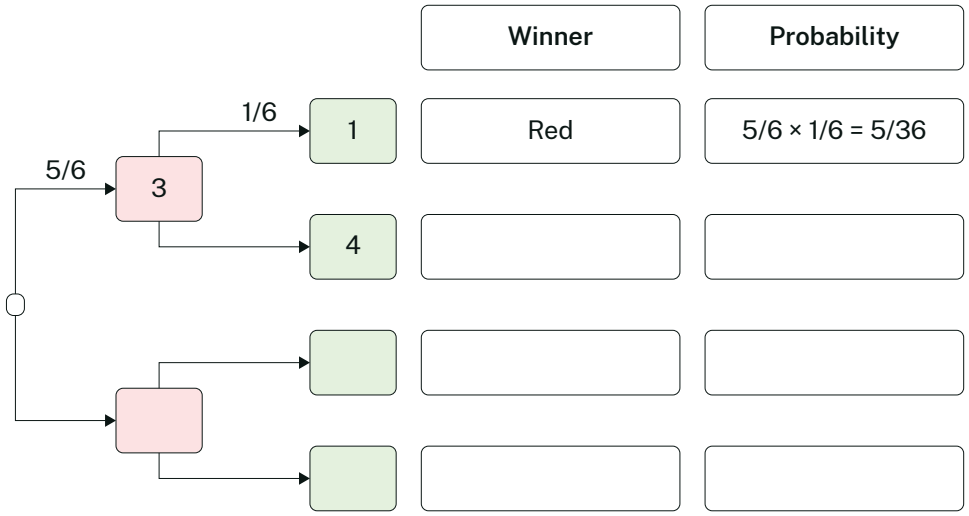
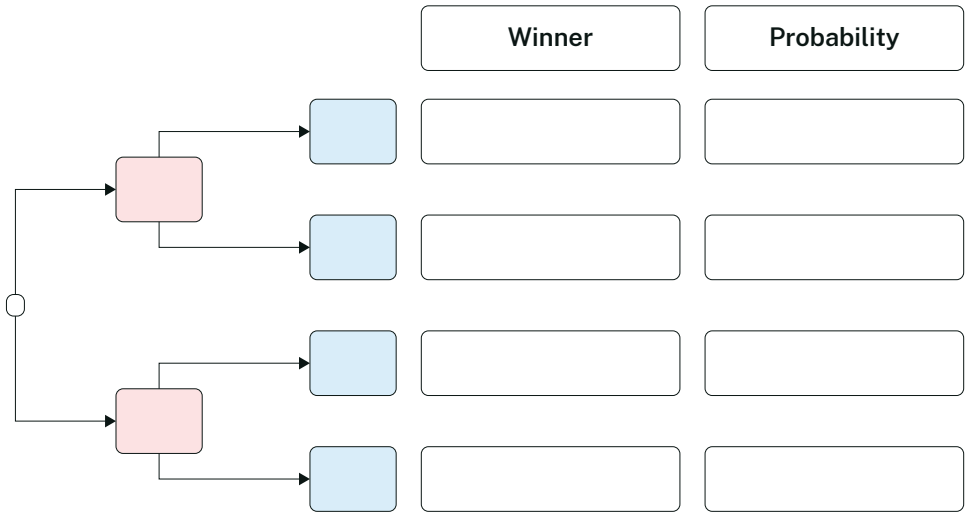
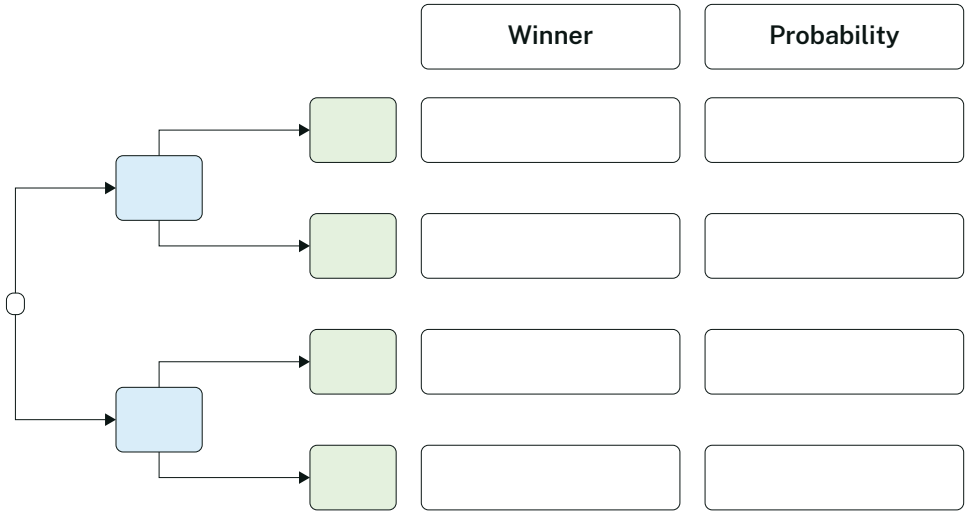






Task 4

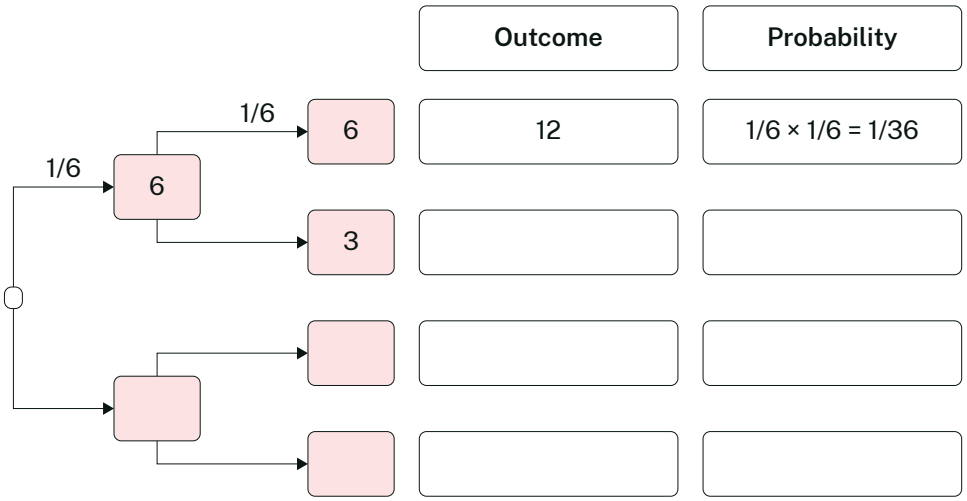
Write down your observations of these outcomes.

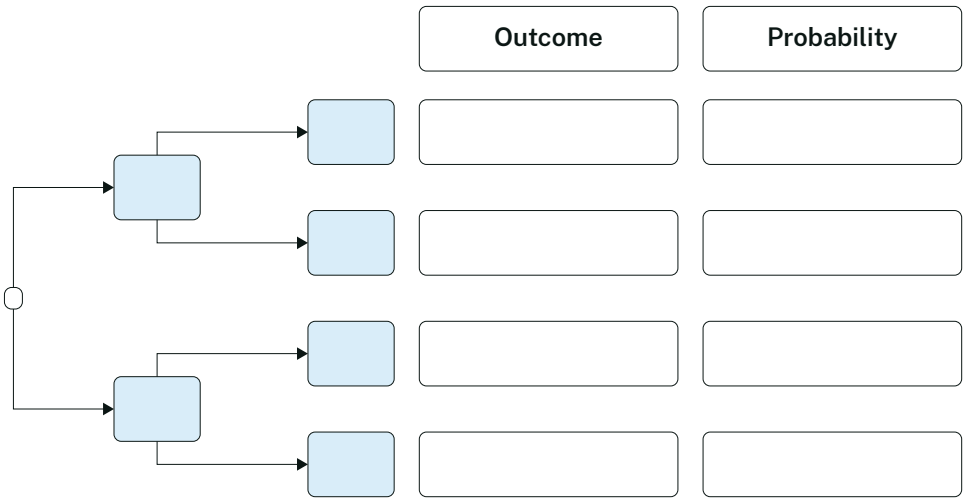
Task 5

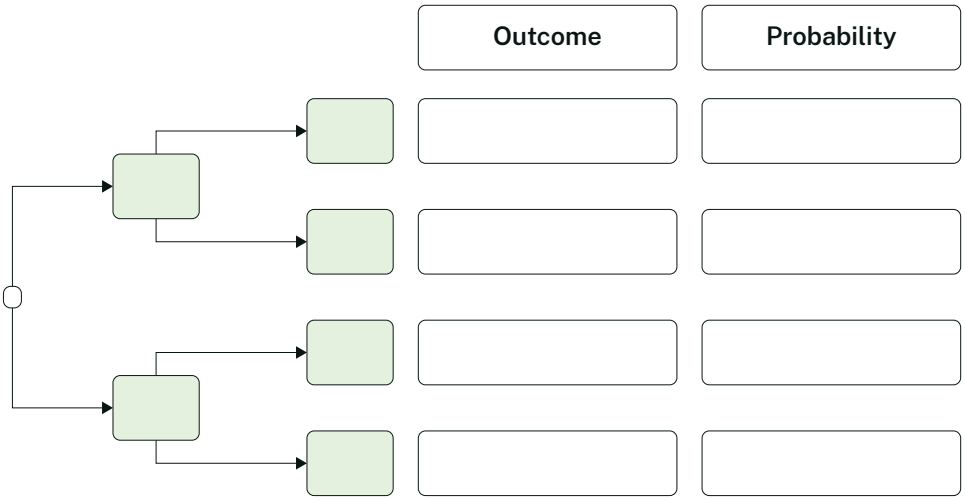
What would happen if the rules of the game changed so that you rolled two dice of each colour?

Create tree diagrams showing the results of rolling two red, two blue, and two green dice.

The first one has been done for you.



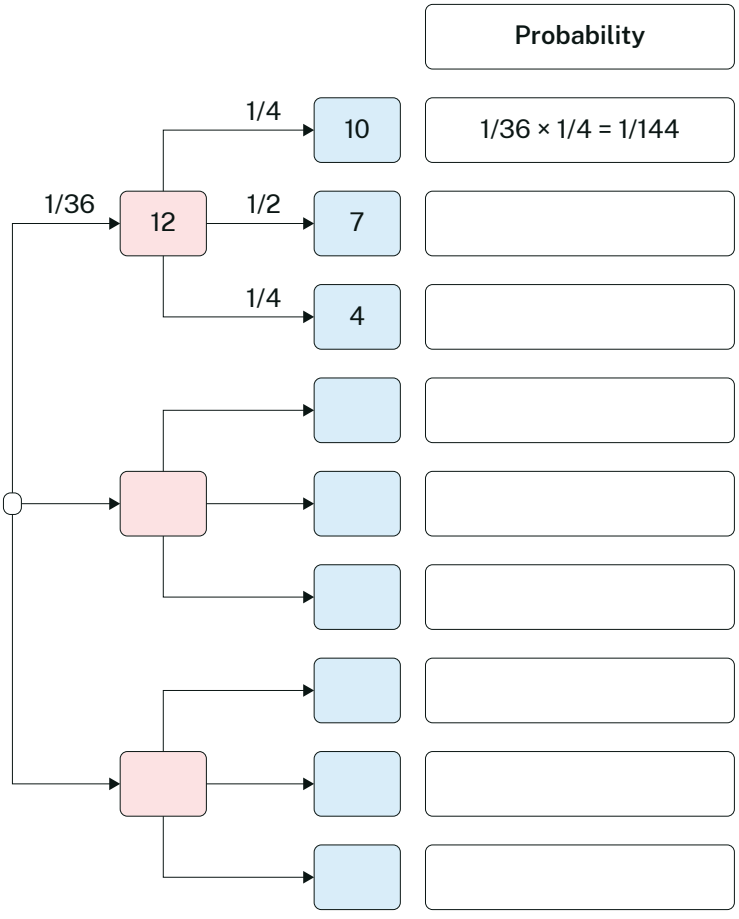




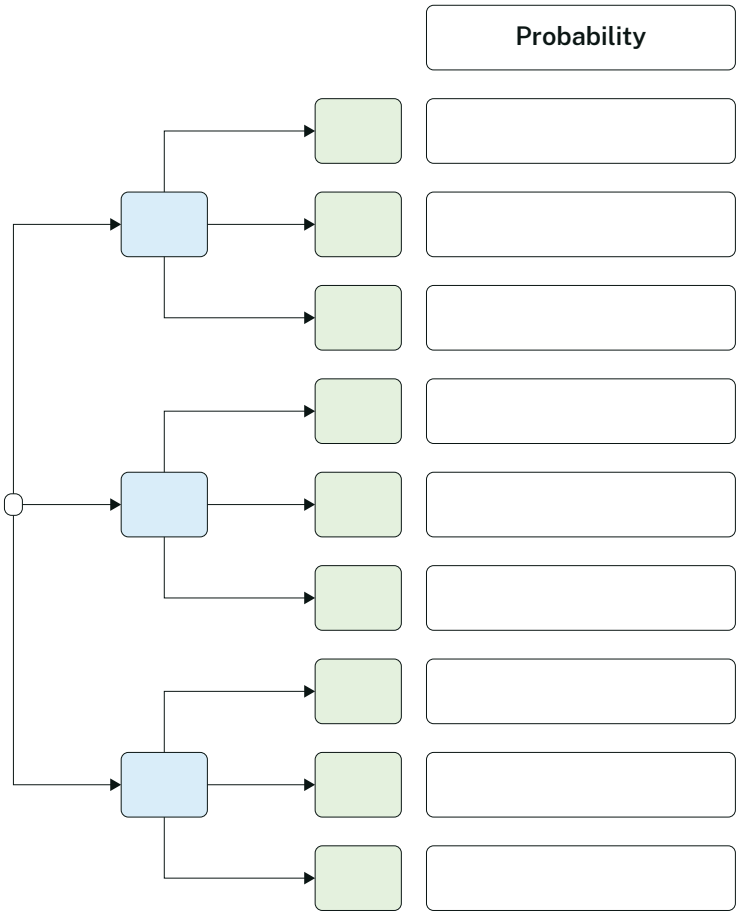
Task 6

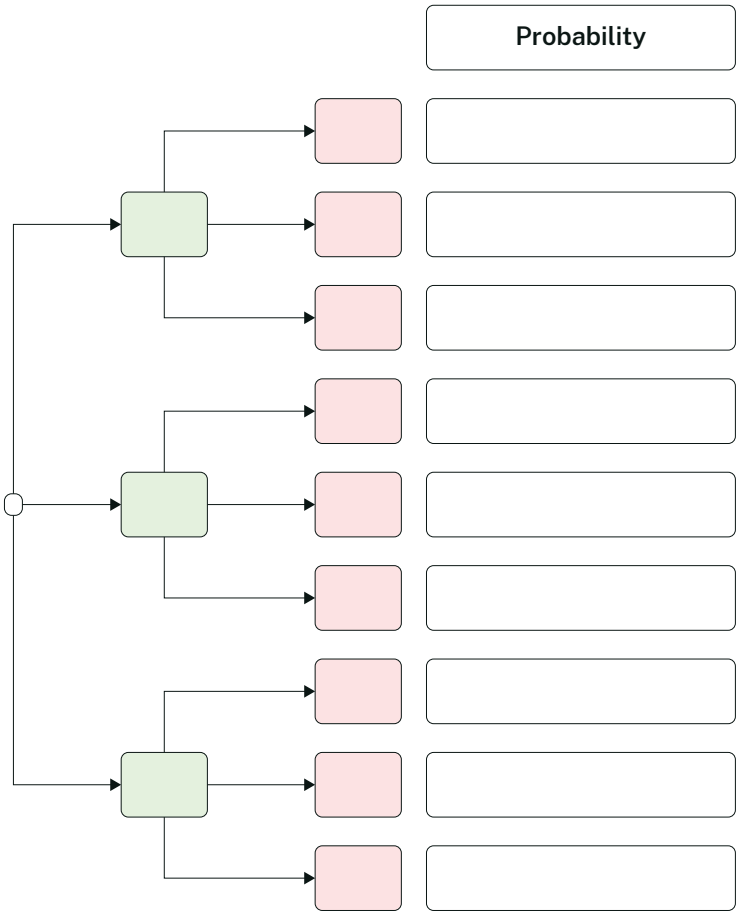
Now that you know the probable outcomes of each double roll, create a final set of three tree diagrams showing the outcomes of a match between each pair of two dice (2 red 2 blue, 2 red 2 green, 2 blue 2 green).

The first one has been done for you.



Appendix A: Student assessment





Task 7

Write down your observations of these outcomes.

Appendix B: Assessment rubric

	Extensive 4	Thorough 3	Sound 2	Basic 1	Not Attempted 0
Application	All tree diagrams were drawn correctly.	Most tree diagrams were drawn correctly.	Some of the tree diagrams were drawn correctly.	An attempt was made to draw the correct tree diagrams.	No attempt at drawing tree diagrams was made.
Analysis	The probability of each outcome was calculated correctly.	Most of the outcome probabilities were calculated correctly.	Some of the outcome probabilities were calculated correctly.	An attempt was made to calculate the probability of each outcome.	No attempt was made to calculate the probability of each outcome.
Evaluation	The observations were made in a thoughtful and mathematical way.	The observations were made in a mostly thoughtful and mathematical way.	The included observations were basic, but reasonable.	An attempt was made to include observations of the two dice games.	No attempt was made to include observations.